Section 2.8 Related Rates

Find the rate of change for $y\left(\frac{dy}{dt}\right)$ for the following function.

$y = x^2 + 3$ When $x = 1$, $\frac{dx}{dt} = 2$

$$\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = 2(1)(2)$$

$$\boxed{\frac{dy}{dt} = 4}$$

A pebble is dropped into a calm pool resulting in ripples. The outer radius, r, is increasing at a constant rate of 1ft/sec. When the radius is 4 feet, what rate is total area changing by?

$\frac{dr}{dt} = 1$

$r = 4$

$\frac{dA}{dt}$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(4)(1)$$

$$\boxed{\frac{dA}{dt} = 8\pi}$$

When radius is 4 my area is increasing at a rate of 8π.

Statement	Math Model
The velocity of a car after traveling for 1 hour is 50 mph	x = miles traveled $\frac{dx}{dt} = 50$, $t = 1$
Water is being pumped into a pool at a rate of 10 ft^3/min	V = volume of water $\frac{dV}{dt} = 10$
A population of bacteria is increasing at a rate of 2000 per hour	b = bacteria at given time $\frac{db}{dt} = 2000$
Revenue is increasing at a rate of \$4000 per month.	R = revenue at given time $\frac{dR}{dt} = 4000$

Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius when r=2.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4.5 = 4\pi(2)^2 \frac{dr}{dt}$$

$$\frac{4.5}{4\pi(2)^2} = \frac{dr}{dt}$$

$$\frac{4.5}{16\pi} = \frac{dr}{dt}$$

$$0.09_{in} = \frac{dr}{dt}$$

A company's profit, P (in dollars), for selling x units is modeled by

$$P = 500x - \left(\frac{1}{4}\right)x^2$$

The sales are increasing at a rate of 10 units per week, what is my profit changing by when x = 500?

$\frac{dx}{dt}$

$$\frac{dP}{dt} = 500\frac{dx}{dt} - \left(\frac{1}{2}\right)x\frac{dx}{dt}$$

$$\frac{dP}{dt} = 500(10) - \left(\frac{1}{2}\right)(500)(10)$$

$$\boxed{\frac{dP}{dt} = \$2500 \text{ per week}}$$